NUMERICAL MODELING OF THE PROCESS OF HEATING A PETROLEUM STRATUM BY MEANS OF HIGH-FREQUENCY ELECTROMAGNETIC RADIATION

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One of the most promising methods of intensifying the production yield of high-viscosity petroleum and gaseous hydrates, as well as overcoming complications in wells, associated with the settling out of paraffins and gaseous hydrates, involves the utilization of high-frequency electromagnetic radiation [1-4]. Owing to its deep penetration and the resulting internal liberation of heat electromagnetic radiation is capable of attaining a far higher rate of heating and uniformity in the latter at a higher efficiency than is possible with traditional heating methods involving steam or hot liquid. However, in order to realize these potentials it is necessary, in detail, to study the processes of heat and mass transfer which occur under these conditions in order to find the optimum operational regimes. The initial theoretical estimates of the depth of heating and of the distribution of temperatures in the heated zone were undertaken in [1]. These estimates were subsequently refined in [2-4]; however, no studies were undertaken, and none exist up to the present time, involving two-dimensional models. At the same time, at a stratum thickness of $H \sim 1$ m, a radiation penetration depth of $\ell \sim 10$-100 m, and a heating time on the order of tens and hundreds of hours, the loss of heat to the rocks adjacent to this stratum, both above and below, will be significant. As a consequence of this loss of heat a steady temperature distribution is established over time (whereas in a one-dimensional axisymmetric model, in the presence of a constant power source at the coordinate origin, the temperature throughout the entire region will grow without limit over time).

The dimensions of the heated zone, determined from the depth of penetration of some fixed isotherm, given a source of fixed power and stratum thickness, depend on the thermophysical parameters of the medium and on the absorption factor $\alpha = 1/\ell$. The latter, in turn, depends on the frequency of the electromagnetic radiation, and consequently this factor can be controlled. With small α (a great depth ℓ of radiation penetration) the energy from the source is scattered over a broad region and is dissipated into the adjacent rocks, without achieving the required heating. With large α (a low depth ℓ) strong heating of a small area surrounding the source results and this leads to a significant gradient of temperature, with the heat moving intensively above and below the adjacent rocks, again not providing the required radial heating. In each of these cases the heated zone is small and the heating ineffective. Consequently, there exists some optimum absorption factor at which (given a fixed power source) that the greatest dimensions for the heated zone can be achieved. It is obvious that there also exists an optimum heating time (for each value of α) at which the ratio of the volume of the heated zone relative to the energy expended is at a maximum.

It is the purpose of the present study to determine these optimum parameters, as well as to refine the other quantities which characterize the process of high-frequency stratum heating.

Model and a System of Equations. The investigation was conducted on a two-dimensional axisymmetric model such as that shown in Fig. 1. A petroleum stratum of thickness H is contained between planes perpendicular to the z axis (the upper plane is identified as line 3). The stratum is surrounded, both above and below, by an unbounded medium whose thermophysical characteristics differ from those of the stratum. Source 1 with a power of several tens or hundreds of kilowatts, emitting electromagnetic waves in the radial direction, is immersed into a well whose surface is denoted by line 2. As a consequence of the volumetric absorption of electromagnetic energy around the well, the stratum and the adjacent rock are subjected to heating. Curves 4-8 are isotherms of the temperature fields at a particular instant in time (see below).

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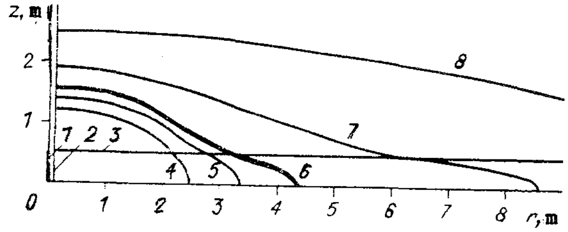

Fig. 1

The electromagnetic radiation absorption factor α is assumed to be constant, i.e., independent of time, coordinate, and temperature, so that the absorption of the radiation follows the Bouguer–Lambert law. However, as was stated earlier, this coefficient does depend on the frequency of the electromagnetic radiation and its magnitude can be established prior to the onset of the heating process. The filtering motion of the petroleum and the related convective transfer of heat, as well as the exchange of heat to the surface of the well, are all neglected.

Within the framework of this model the process involved in the heating of the stratum and the adjacent rocks is described by a two-dimensional equation of heat conduction with a volumetric source

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\lambda r \frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) + \frac{\alpha W \psi(z)}{2\pi r}\exp[\alpha(b - r)],$$
$$b \leqslant r < \infty, \quad -\infty < z < \infty, \tag{1}$$

where the density ρ, the heat capacity c, and the thermal conductivity λ are different within the stratum and in the adjacent rock and, consequently, are functions of z; b is the radius of the well, W represents the linear power of the radiation source (watts per unit length along the z axis), and the function ψ(z) characterizes the distribution, over height, of the absorbed power of the electromagnetic radiation. In the ideal case considered in this study, where the electromagnetic wave "channels" along the stratum without penetrating into the adjacent rocks, the function ψ(z) has the form

$$\psi(z) = \begin{cases} 1 & \text{when } -H/2 \leqslant z \leqslant H/2, \\ 0 & \text{when } z < -H/2,\ z > H/2. \end{cases} \tag{2}$$

The last term in formula (1) expresses the density of the volumetric evolution of heat which comes about as a consequence of the absorption of the electromagnetic radiation. Indeed, the change in the radiant intensity I along the coordinate r can be represented as

$$dI = -(1/r)I dr - \alpha I dr, \tag{3}$$

where the first term expresses the reduction in intensity as a consequence of the geometric dispersion of the radiation, while the second term expresses this phenomenon as a consequence of absorption. As a result of the integration of (3) in the assumption that α = const and with consideration of the fact that the intensity $I_0 = W/2\pi b$ at the surface of the well, we have

$$I = \frac{W}{2\pi r}\exp[\alpha(b - r)], \quad r \geqslant b;$$

the product αI(r) yields the den ,ity of the volumetric evolution of heat.

The process of paraffin melting or the decomposition of the gaseous hydrate is accounted for in the following manner. It is assumed that the heat capacity c within the stratum exhibits a singularity at the phase transition temperature T_s:

$$c(T) = c_0 + L\delta(T - T_s) \tag{4}$$

(L is the latent heat of phase transition and δ represents the delta function, which in numerical calculations is replaced by a "step" of finite width $2\Delta T_s$ [5]). Since the heat-capacity values for temperatures below and above T_s are different (c_0 and c_1, respectively), we can write the function c(T) in the form

$$c(T)=\begin{cases}c_0 & \text{when } T<T_s-\Delta T_s,\\ (c_0+c_1)/2+L/2\Delta T_s & \text{when } T_s-\Delta T_s\leqslant T\leqslant T_s+\Delta T_s,\\ c_1 & \text{when } T>T_s+\Delta T_s.\end{cases} \quad (5)$$

Thus, the problem under consideration here is essentially nonlinear, and it can be included among the class of Stefan problems. Its boundary conditions are as follows:

$$\left.\frac{\partial T}{\partial r}\right|_{r=b}=0,\quad \left.\frac{\partial T}{\partial r}\right|_{r\to\infty}\to 0,\quad \left.\frac{\partial T}{\partial z}\right|_{z\to\pm\infty}\to 0. \quad (6)$$

For a numerical solution of Eq. (1) it is convenient to bring it to dimensionless form. If we take the depth of radiation penetration $\ell = 1/\alpha$ as the unit of length, and if we take the phase transition temperature T_S as the unit of temperature, then (1) can be written in the following dimensionless form:

$$C\frac{\partial\theta}{\partial\tau}=\frac{1}{x}\frac{\partial}{\partial x}\left(\Lambda x\frac{\partial\theta}{\partial x}\right)+\frac{\partial}{\partial\zeta}\left(\Lambda\frac{\partial\theta}{\partial\zeta}\right)+\frac{Q}{x}\exp(\beta-x) \quad (7)$$

$$(x=r\alpha,\ \zeta=z\alpha,\ \beta=b\alpha,\ \tau=\alpha^2a_0t,\ \theta=T/T_s);$$

$$Q=W\psi(z)/2\pi\lambda_0T_s,\ C=\rho c/\rho_0c_0,\ \Lambda=\lambda/\lambda_0,\ a_0=\lambda_0/\rho_0c_0, \quad (8)$$

ρ_0 and λ_0 are the density and thermal conductivity of the stratum.

Method of Solution. To solve the formulated problem we employed the implicit method of variable directions with iterations based on the nonlinear dependence of the heat capacity on temperature (5) in a nonuniform grid [5, 6].

According to this method, the solution of the two-dimensional equation (7) is sought through successive solution of one-dimensional problems along x and ζ. The interval $\Delta\tau$ is broken down into two half steps, and Eq. (7) is transformed into two equations solved successively:

$$C_{ij}\frac{\theta_{ij}^{k+1/2}-\theta_{ij}^{k}}{0{,}5\Delta\tau}=\frac{\Lambda_jx_{i+1}\left(\theta_{i+1,j}^{k+1/2}-\theta_{ij}^{k+1/2}\right)/\Delta x_i-\Lambda_jx_i\left(\theta_{i,j}^{k+1/2}-\theta_{i-1,j}^{k+1/2}\right)/\Delta x_{i-1}}{x_i\Delta x_i}+$$
$$+\frac{\Lambda_{j+1}\left(\theta_{i,j+1}^{k}-\theta_{ij}^{k}\right)/\Delta\zeta_j-\Lambda_j\left(\theta_{ij}^{k}-\theta_{i,j-1}^{k}\right)/\Delta\zeta_{j-1}}{\Delta\zeta_j}+\frac{Q}{x_i}\exp(\beta-x_i) \quad (9)$$

which is the first half step and

$$C_{ij}\frac{\theta_{ij}^{k+1}-\theta_{ij}^{k+1/2}}{0{,}5\Delta\tau}=\frac{\Lambda_jx_{i+1}\left(\theta_{i+1,j}^{k+1/2}-\theta_{ij}^{k+1/2}\right)/\Delta x_i-\Lambda_jx_i\left(\theta_{ij}^{k+1/2}-\theta_{i-1,j}^{k+1/2}\right)/\Delta x_{i-1}}{x_i\Delta x_i}+$$
$$+\frac{\Lambda_{j+1}\left(\theta_{i,j+1}^{k+1}-\theta_{ij}^{k+1}\right)/\Delta\zeta_j-\Lambda_j\left(\theta_{ij}^{k+1}-\theta_{i,j-1}^{k+1}\right)/\Delta\zeta_{j-1}}{\Delta\zeta_j}+\frac{Q}{x_i}\exp(\beta-x_i) \quad (10)$$

which is the second half step.

Each of these equations is solved by a sweeping method in the corresponding direction: (9) is solved in the direction of x ($\theta_{ij}{}^{k+1/2}$ is found from the known $\theta_{ij}{}^{k}$), (10) is solved in the direction of ζ ($\theta_{ij}{}^{k+1}$ is determined from the known $\theta_{ij}{}^{k+1/2}$). Here, in each half step an iteration process is constructed in which the relationship between C and temperature is taken into consideration, the temperature having been specified in formula (5).

An ES FORTRAN program was compiled for this algorithm. The calculations were carried out on an ES 1045 computer. This involved a 50 × 50 grid with the interval, on increasing distance along both coordinates from the source growing in a manner such that the dimensions of the area covered by the grid were considerably greater than 1 along both coordinates, thus ensuring satisfaction of boundary conditions (6). The initial temperature of the medium was assumed to be constant.

Results and Discussion. The results of the calculations are presented in Figs. 1-4 in dimensional form, for the sake of clarity, since there are numerous dimensionless parameters (Λ, C, Q, β, $L/2\Delta T_Sc_0$) and their utilization offers little in the form of generality

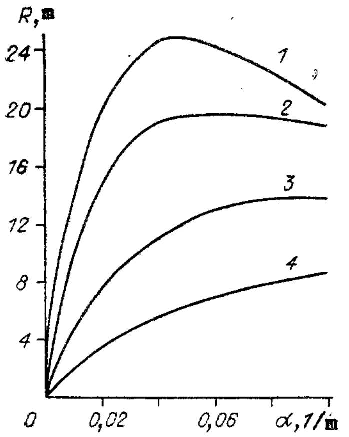

Fig. 2

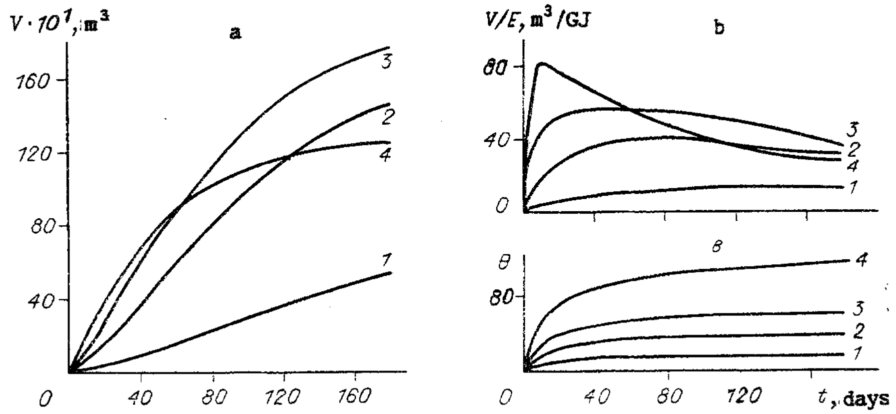

Fig. 3

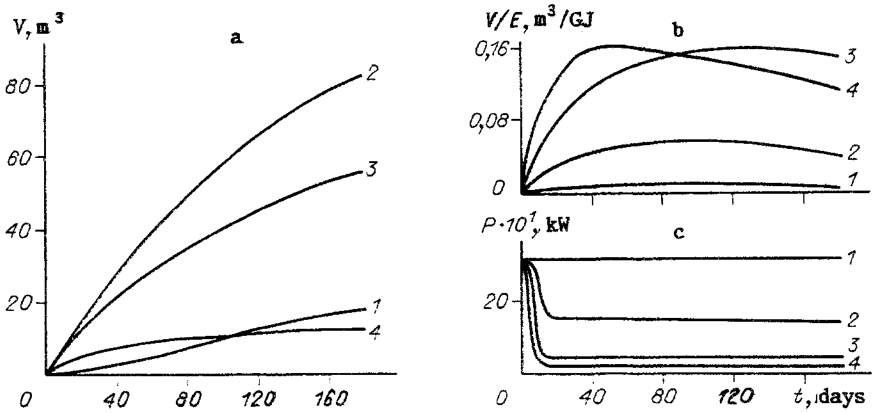

Fig. 4

in the presentation of the results. We used the values of parameters characteristic for materials in petroleum technology: in the stratum $\rho_0 = 10^3$ kg/m^3, $\lambda_0 = 1$ W/(m·K), $c_0 =$ 2.1 kJ/(kg·K), L = 160 kJ/kg, $T_S = 50$°C, and in the adjacent rocks $\rho = 2 \cdot 10^3$ kg/m^3, $\lambda =$ 2.4 W/(m·K), c = 0.8 kJ/(kg·K). The absorption factor varied within limits of 0.01-0.1 1/m, which is characteristic of the electromagnetic waves in the meter and decimeter range. The thickness of the stratum was 1 m, and the power of the source was 315 kW. Recalculation of the results for the other values of the parameters can be easily accomplished with the aid of (8).

Figure 1, which was used earlier to describe the model, shows the characteristic form of the temperature-field isotherms ($\alpha = 0.01$ 1/m, after 25 days following the onset of heating): curve 4 illustrates the 100°C isotherm; curve 5 shows the isotherm for 70°C, and 6 represents 50°C (the melting front), 7 represents 30°C, and 8 reflects the 10°C isotherm.

The distance R which is covered by the melting front along the r axis depends, as was stated earlier, on the absorption factor α. This relationship is shown in Fig. 2 for the instants of time 180, 90, 30, and 10 days (curves 1-4). A heating time of 180 days, although inadequate to establish a steady temperature field, is large from the practical point of view; longer periods of heating are of no interest. The calculations confirm the estimate, carried out earlier in [1-4], of the maximum heating radius of several tens of meters. Curve 4 in Fig. 2 exhibits a clear maximum at $\alpha = 0.05$ 1/m (the optimum value). The physical sense of this optimum will be discussed in the following.

Figure 3 illustrates the dynamics of the most important parameters in the stratum heating process for various values: a) the increase in the volume V(t) of the melt zone in the stratum; b) the ratio of the melted volume V to the energy E emitted by the source at the onset of heating; c) the rise in temperature $\theta_0 = (T - T_0)/(T_S - T_0)$ (T_0 is the initial temperature of the medium) at the well surface at the point z = 0, exhibiting the highest temperature. Curves 1-4: $\alpha = 0.01, 0.03, 0.05, 0.1$ 1/m. With the maximum $\alpha = 0.1$ 1/m the melting front initially moves rapidly, but subsequently its motion is decelerated, since the depth of penetration ($\ell = 10$ m) for the electromagnetic waves is small and the appearance of large ℓ deep within the stratum comes about primarily as a result of heat conduction. With minimum $\alpha = 0.01$ 1/m the energy from the source is distributed to a somewhat large volume and the heating proceeds slowly. The optimum value for $\alpha = 0.05$ 1/m is achieved, as stated earlier, by the largest R and V with rather prolonged heating times. The V/E curves characterize the efficiency of the heating process from another point of view, namely the attainment of the largest melted volume for the least expenditures of energy, and they exhibit clear maxima which define the optimum (from this point of view) heating times for given α. The maximum of the V/E ratio is reached at the highest α; however, the absolute value of V in this case, as can be seen from Fig. 3, is not large. Thus, the data from Fig. 3 allow us to select the most expedient heating technology for specific practical problems and conditions.

With continuous heating a large melting radius can be achieved, as well as a large melt zone; however, the well surface in this case (see Fig. 3c) is strongly heated, i.e., its temperature reaches hundreds and thousands of degrees. It should be noted that under real conditions the temperature will be lower, since strong heating without an influx of oxygen will lead to decomposition of the petroleum near the well, the removal of its light components and, consequently, to additional losses of heat. However, in the model under consideration no consideration has been given to these processes. Where strong heating is unacceptable, we have some interest in modeling such a heating regime, where the temperature of the well surface does not exceed some specified value, which might be attained through periodic disconnecting and connecting of the source or smooth reduction of the power after the specified maximum temperature has been reached at the point z = 0. The depth of the melting in this case, of course, proves to be substantially lower. The dynamics of the heating process in such a regime, given a maximum well surface temperature of $\theta_0 = 4.2$, is shown in Fig. 4. The significance of Fig. 4a, b is the same as in Fig. 3a, b, while Fig. 4c shows the manner in which the temperature at the surface, on reaching values of $\theta_0 = 4.2$, will remain at this level in the future. Curves 1-4 represent $\alpha = 0.002, 0.01, 0.1$, and 1.0 1/m. As we can see from Fig. 4, the motion of the melt front depends less strongly on α, and in the calculations we have therefore taken a broader range of values for α. The heating efficiency, i.e., the V/E ratio (Fig. 4b), is higher for large α, since the same depth of melting is achieved with a lower average power (Fig. 4c), owing to the fact that with small α a considerable portion of the energy is expended on useless heating of the area $r \gg R$. The V/E curves in Fig. 4b, just as in Fig. 3b, exhibit maxima which determine the optimum heating time.

Conclusions. We have completed a numerical study on a two-dimensional model in which consideration has been given to the nonlinear heat-capacity function c(T) of the heating process for a petroleum stratum. It has been demonstrated that the efficiency of the heating depends significantly on correct selection of the parameters, and we have particular reference here to the absorption factor α defined by the radiation frequency. We have determined the optimum values of α, as well as the optimum heating times. We have carried out calculations to confirm the possibility of utilizing high-frequency electromagnetic heating of petroleum strata in order to intensify the production yield of high-viscosity petroleum and to combat other well problems, and these can also be utilized for the development of practical recommendations.

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AXISYMMETRIC FUNDAMENTAL SOLUTIONS FOR THE EQUATIONS OF HEAT CONDUCTION IN THE CASE OF CYLINDRICAL ANISOTROPY OF A MEDIUM

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UDC 517.95

Numerical methods of solving the boundary-value problems for the equations of mathematical physics based on the application of fundamental solutions, i.e., solutions describing the reaction of infinite space or an infinite plane to a concentrated action, are currently in widespread favor. Among these methods we can include the direct and indirect methods of boundary integral equations [1], as well as the method of sources in which the solution of the boundary-value problem is constructed by superposition of concentrated actions in space, above some surface encompassing the area under investigation [2]. For the equations of steady and nonsteady heat conduction in an isotropic medium such solutions are well established (see [1] and the references cited there) both for the two- and three-dimensional cases, as well as for the case of the axisymmetric problem. The plane and three-dimensional equations of heat conduction for a rectilinear anisotropic medium can be reduced to the isotropic case. We know of three-dimensional fundamental solutions for the equations of elasticity theory in the case of a medium with rectilinear anisotropy [3] and for a rectilinear anisotropic hereditary (or memory) elastic medium [4, 5].

The axisymmetric fundamental solutions for the steady and nonsteady equations of heat conduction in the case of a cylindrical anisotropic medium are constructed in the present study by reducing them to the corresponding equations for isotropic media. We present the limit relationships for the characteristic parametric values. As one of the limit cases we have derived the fundamental solutions for the steady and nonsteady equations of plane heat-conduction problems for a rectilinear anisotropic medium.

The equations of nonsteady heat conduction in an arbitrarily anisotropic medium have the form

$$\operatorname{div} \mathbf{q} + cT_{,t} = Q, \quad \mathbf{q} = -\chi \nabla T. \tag{0.1}$$

Here T is the temperature; $\mathbf{q}$ is the heat-flux vector; Q is the specified release of heat; c is the coefficient of heat capacity; χ is the symmetric heat-conduction tensor; t is time. The subscript which appears after the comma denotes the derivative with respect to the cor-

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